The Game's Afoot

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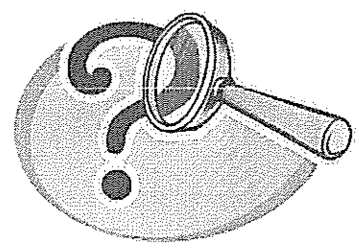
The Case of the Cooling Corpse

It was a dark and stormy night. Holmes and Watson were called to the scene of the murder by Inspector Lestrade of the police. The victim was a wealthy but cruel man. He had many enemies.

The most likely suspects are the wife, the business partner, and the butler. Each has an equally strong motive. Each also has an alibi. The wife claims to have spent the entire evening at the theater across town. She was seen leaving the theater at 10:30 p.m. and returned home at 11:00 p.m., going straight up to her bedroom. Her return was verified by the upstairs maid. The business partner claims to have spent the evening working on papers at the office. His wife and household staff verified that he returned home at 10:30 p.m. The butler was on his night off. He claims to have been at the local pub until 10:00 p.m. The butler returned to his quarters above the carriage house at 10:05 p.m. and did not leave. This was verified by the other servants.

The body was found in the victim's study. Holmes arrived at the scene at 4:30 a.m. The room was unusually warm and stuffy. One of the police officers went to open a window. Holmes admonished him to delay that action until he had completed his investigation of the crime scene. He instructed Watson to determine the temperature of the body. This was found to be 88.0°. Holmes questioned the servants as to the room temperature during the evening and learned that the man had liked the room warm and that the temperature in the study was always very near the current 76°. Holmes asked Watson to take the temperature of the body again at the conclusion of his inspection of the scene, two hours after the first reading. It was 85.8°.

Figure out "who did it" by applying Newton's Law of Cooling.



$$u(t) = T + (u_0 - T)e^{-kt}$$

Your Job as a group:

1) Figure out "who did it" by applying Newton's Law of Cooling.

Newton's Law of Cooling states:

$$u(t) = T + (u_0 - T)e^{-kt}$$

In this formula, T represents the temperature of the surrounding area, u_0 is the initial temperature of the object being analyzed, t is the length of time in hours, k is the rate at which the body is cooling, and $u(t)$ represents the temperature after time t. Use 98.6° as the normal body temperature

2) After solving the mystery, write a CLAIM as to who committed the murder. Provide EVIDENCE (in terms of equations and/or graphs) to support your claim. Then provide a REASON for why that mathematics supports your claim (you can also add in a fictitious motive for the killer if you have time).

3) Answer the questions on the back of this task sheet related to the mathematics in this lesson.

Questions:

1.) What function were you considering to help you find the murderer?

$u(t) = 76 + (88-76)e^{-.101t}$

2.) What is the y-intercept of the function you used to catch the murder? What does it represent?

(0, 88) At the time Holmes found the body, the temperature was 88°F.

3.) What is the domain of the function as written? What would the domain be in real life?

As written: D: $(-\infty, \infty)$

In real life, he wouldn't be warmer than 98.6 so D: $(-6.26, \infty)$

4.) What is the range of the function as written? What would be the range in real life?

R: $(76, \infty)$ as written

In real life, he wouldn't be warmer than 98.6 so R: $(76, 98.6]$

5.) Where is the horizontal asymptote?

a. Why (in the context of the problem) does this appear?

Because when objects cool off, they cool to match their surroundings then stop. The body would cool until it reached 76°.

b. Why (mathematically) would there be an asymptote here?

$e^{-\text{anything}} = \dfrac{1}{e^{\text{anything}}}$ - When e is raised to a very large number, then $\dfrac{1}{e^{\text{large number}}}$ approaches zero.

- Then $(88-76)e^{-.101t}$ approaches zero as t gets large.

6.) Identify the intervals of increase and decrease in the function.

Function is decreasing on $(-\infty, \infty)$

7.) If the room temperature was a bit cooler, say 72 degrees, what would change on the graph?

The horizontal asymptote would be at 72°.

8.) What would change about your graph if the constant (k) was positive instead of negative?

The graph would increase instead (flip over y axis).