Honors PreCalculus
Unit 8 Review #1

Name_______________

Simplify.

1. $\cot x \tan x$

$\frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x} = 1$

2. $\frac{1-\cos^2 x}{\sin x}$

$\frac{\sin^2 x}{\sin x} = \sin x$

3. $\sec(-x)\cos(-x)$

$-\sec(x)(-\cos(x))$

$\sec(x)\cos(x)$

$\boxed{1}$

4. $\sin\theta - \tan\theta\cos\theta + \cos\left(\frac{\pi}{2}-\theta\right)$

$\sin\theta - \frac{\sin\theta}{\cos\theta}\cos\theta + \sin\theta$

$\boxed{\sin\theta}$

5. $\frac{\sin\left(\frac{\pi}{2}-x\right)\cot x}{\cos^2 x}$

$\frac{\cos x \cot x}{\cos^2 x}$

$\frac{\frac{\cos x}{\sin x}}{\cos x} = \frac{1}{\sin x}$

$\boxed{\csc x}$

6. $\frac{1}{1-\sin x} + \frac{1}{1+\sin x}$

$\frac{(1+\sin x)+(1-\sin x)}{1-\sin^2 x}$

$\frac{2}{\cos^2 x} = \boxed{2\sec^2 x}$

7. $1-2\sin x+\sin^2 x$

Don't do

Find the exact value using sum and difference.

8. $\cos 75°$

$\cos(30+45) =$

$\cos 30 \cos 45 - \sin 30 \sin 45$

$\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

9. $\sin\frac{7\pi}{12} = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$

$\frac{\sqrt{2}}{2}\frac{1}{2} + \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}$

$\boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

Write as a single expression.

10. $\cos 7y \cos 3y - \sin 7y \sin 3y$

$\cos(7y+3y) = \boxed{\cos(10y)}$

11. $\frac{\tan 3\alpha - \tan 2\beta}{1 + \tan 3\alpha \tan 2\beta}$

$\boxed{\tan(3\alpha - 2\beta)}$

Find all solutions between the interval $[0,2\pi]$

12. $\sin 2x = \sin x$

$2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \quad 2\cos x - 1 = 0$

$\cos x = 1/2$

$\boxed{x = 0, \pi \quad \frac{\pi}{3}, \frac{5\pi}{3}}$

13. $\cos 2x + \sin x = 0$

$1 - 2\sin^2 x + \sin x = 0$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -1/2 \quad \sin x = 1$

$\boxed{\frac{7\pi}{6}, \frac{11\pi}{6} \quad \frac{\pi}{2}}$

Use half-angle identities to find the exact value.

14. $\cos\frac{\pi}{8}$

$\cos\left(\frac{2\pi/8}{2}\right) = \cos\left(\frac{\pi/4}{2}\right)$

$x = \pi/4$

$\cos\left(\frac{\pi/4}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$

$= \pm\sqrt{\frac{1+\cos(\pi/4)}{2}}$

$= \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \boxed{\pm\sqrt{\frac{2+\sqrt{2}}{4}}}$

15. $\sin\frac{\pi}{8} = \sin\left(\frac{2\pi/8}{2}\right) = \sin\left(\frac{\pi/4}{2}\right)$

$\sin\frac{\pi/4}{2} = \pm\sqrt{\frac{1-\cos(\pi/4)}{2}}$

$= \pm\sqrt{\frac{1-\sqrt{2}/2}{2}}$

$\boxed{= \pm\sqrt{\frac{2-\sqrt{2}}{4}}}$

Proof Review

1. $\sec^2 x \csc^2 x = \sec^2 x + \csc^2 x$

$$\text{RHS} \quad \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$$

$$\Rightarrow \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} = \frac{1}{\sin^2 x \cos^2 x}$$

$$\Rightarrow \frac{1}{\sin^2 x} \bullet \frac{1}{\cos^2 x} = \sec^2 x + \csc^2 x$$

□

2. $\sec x(\sec x - \cos x) - \tan^2 x = 0$

$$\sec x(\sec x - \cos x) - (\sec^2 x - 1)$$

$$\cancel{\sec^2 x} - \sec x \cos x - \cancel{\sec^2 x} + 1$$

$$-\sec x \cos x + 1$$

$$-\frac{1}{\cos x} \cdot \cos x + 1$$

$$-1 + 1 = 0$$

□

3. $\dfrac{1-\sin x}{\cos x} = \dfrac{\cos x}{1+\sin x}$

RHS

$$\frac{\cos x}{1+\sin x} \frac{(1-\sin x)}{(1-\sin x)} = \frac{\cos x(1-\sin x)}{1-\sin^2 x}$$

$$\frac{\cos x(1-\sin x)}{\cos^2 x} = \frac{1-\sin x}{\cos x}$$

□

4. $2\csc x = \dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x}$

RHS

$$\frac{\sin^2 x}{\sin x(1+\cos x)} + \frac{(1+\cos x)^2}{\sin x(1+\cos x)}$$

$$\frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1+\cos x)}$$

KING

$$\frac{2 + 2\cos x}{\sin x(1+\cos x)} = \frac{2(1+\cos x)}{\sin x(1+\cos x)} = \frac{2}{\sin x}$$

$$\Rightarrow \boxed{2\csc x}$$

5. $\dfrac{\sin^2\theta - \tan\theta}{\cos^2\theta - \cot\theta} = \tan^2\theta$

$$\frac{\sin^2\theta - \frac{\sin\theta}{\cos\theta}}{\cos^2\theta - \frac{\cos\theta}{\sin\theta}} = \frac{\frac{\sin^2\theta\cos\theta - \sin\theta}{\cos\theta}}{\frac{\cos^2\theta\sin\theta - \cos\theta}{\sin\theta}}$$

$$\frac{\sin^2\theta\cos\theta - \sin\theta}{\cos\theta} \cdot \frac{\sin\theta}{\cos^2\theta\sin\theta - \cos\theta} = \frac{\sin^2\theta(\cancel{\cos\theta-1})}{\cos^2\theta(\cancel{\sin\theta-1})}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$

□

6. $\cos\beta = \sec\beta - \sec\beta\sin^2\beta$

$$\text{RHS} \quad \sec\beta(1-\sin^2\beta)$$

$$\sec\beta(\cos^2\beta)$$

$$\frac{1}{\cos\beta}(\cos^2\beta)$$

$$\cos\beta$$

□

7. $\sin^2 x\sec^2 x - \sec^2 x = -1$

$$\sec^2 x(\sin^2 - 1)$$

$$\sec^2 x(-\cos^2 x)$$

$$\frac{1}{\cos^2 x}(-\cos^2 x)$$

$$= \boxed{-1}$$

8.* $\tan^2 x + \dfrac{\cos x}{1+\sin x} = \sec x$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos x}{1+\sin x}$$

$$\frac{\sin^2 x(1+\sin x) + \cos^3 x}{\cos^2 x(1+\sin x)}$$

9. $\dfrac{1+\cos x}{\sin x} = \csc x + \cot x$

RHS: $\dfrac{1}{\sin x} + \dfrac{\cos x}{\sin x}$

$\dfrac{1+\cos x}{\sin x}$ $\square$

10. $\dfrac{\tan x - 1}{\tan x + 1} = \dfrac{1-\cot x}{1+\cot x}$

RHS: $\cancel{\dfrac{(1-\cot x)^2}{(1+\cot x)(1-\cot x)}}$

$\Rightarrow \dfrac{\cancel{1-2\cot x+\cot^2 x}}{1-\cot^2 x}$

LHS:

11. $8\csc^2\theta - 3\cot^2\theta = 3 + 5\csc^2\theta$

$8\csc^2\theta - 3(\csc^2\theta - 1)$

$8\csc^2\theta - 3\csc^2\theta + 3$

$5\csc^2\theta + 3$ $\square$

12. $\tan x(\cot x + \tan x) = \sec^2 x$

$\tan x \cot x + \tan^2 x$

$1 + \tan^2 x$

$\sec^2 x$ $\square$

13. $\dfrac{1}{\cos x \cot x \tan x} = \sec x$

$\dfrac{1}{\cos x \dfrac{\cos x}{\sin x}\dfrac{\sin x}{\cos x}}$

$\dfrac{1}{\cos x}$

$\sec x$ $\square$

15. $\dfrac{\tan^2 x \cos x}{2\sec x} = \dfrac{1}{2}\sin^2 x$

LHS $\dfrac{\dfrac{\sin^2 x}{\cos^2 x}\cos x}{2\left(\dfrac{1}{\cos x}\right)}$

$\dfrac{\sin^2 x}{\cos x} \cdot \dfrac{\cos x}{2}$

$\Rightarrow \boxed{\frac{1}{2}\sin^2 x}$ $\square$

16. $\dfrac{\tan^2 x - 1}{1+\tan x} = \tan x - 1$

LHS
Mult. by conjugate
Pythag
$\tan^2 x - 1 = -\sec^2 x$

$\dfrac{\tan^2 x - 1\ (1-\tan x)}{(1+\tan x)(1-\tan x)}$

$\dfrac{-\sec^2 x\,(1-\tan x)}{1-\tan^2 x}$

$\dfrac{-\sec^2 x\,(1-\tan x)}{\sec^2 x} = -(1-\tan x) = \tan x - 1$ $\square$

17. $\dfrac{\sin x - \cos^2 x - 1}{\sin x - 1} = \sin x + 2$

Top: $\sin x - (1-\sin^2 x) - 1$

$\dfrac{+\sin^2 x + \sin x - 2}{\sin x - 1}$

$\dfrac{(\sin x + 2)\cancel{(\sin x - 1)}}{\cancel{\sin x - 1}}$

$\sin x + 2$ $\square$