

Unit 8 LT1 I can prove trig identities.

**Only change one side of the problem.
Prove the following identities.

1. $\tan x + \cot x = \sec x \csc x$

$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$ Ratio Identities

$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$ Add Fractions

$\frac{1}{\sin x \cos x}$ Pythag. (King)

$\frac{1}{\sin x} \cdot \frac{1}{\cos x}$ Split fractions
 $\sec x \csc x$ Reciprocal \square

3. $(1 - \tan x)^2 = \sec^2 x - 2 \tan x$

$\frac{1 - 2 \tan x + \tan^2 x}{\sec^2 x - 2 \tan x}$ FOIL
Pythag (Queen) \square

2. $(\sin x)(\cot x + \cos x \tan x) = \cos x + \sin^2 x$

$\sin x \cot x + \sin x \cos x \tan x$ Distribute

$\frac{\sin x \cancel{\cos x}}{\cancel{\sin x}} + \frac{\sin^2 x \cancel{\cos x}}{\cos x}$ Reciprocal Identities

$\cos x + \sin^2 x$ Cancel

\square

4. $\frac{\sec^2 \theta - 1}{\sin \theta} = \frac{\sin \theta}{1 - \sin^2 \theta}$

$\frac{\tan^2 \theta + 1 - 1}{\sin \theta}$ Pythag. (Queen)

$\frac{\tan^2 \theta}{\sin \theta}$ Cancel

$\frac{(\frac{\sin \theta}{\cos \theta})}{\cos \theta}$ Ratio Identity

$\frac{\sin \theta}{\cos^2 \theta}$ Cancel

$\frac{\sin \theta}{\cos^2 \theta} = \frac{\sin \theta}{\cos \theta \cdot 1 - \sin^2 \theta}$ Pythag. (King)

5. $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = 2\csc^2 x$

$\frac{1+\cos x + 1-\cos x}{1-\cos^2 x}$ Add Fractions

$\frac{2}{\sin^2 x}$ Simplify - CLT

$2\csc^2 x$ Reciprocal Identity

6. $(\cos x - \sin x)^2 = 1 - 2\sin x \cos x$

$\cos^2 x - 2\cos x \sin x + \sin^2 x$ FOIL
 $1 - 2\cos x \sin x$ Pythag (King)
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7. $\tan x + \sec x = \frac{\cos x}{1-\sin x} \frac{(1+\sin x)}{(1+\sin x)}$ Conjugate

$\frac{\cos x (1+\sin x)}{1-\sin^2 x}$

Pythag (King) $\frac{\cos x (1+\sin x)}{\cos^2 x}$

~~$\frac{\cos x + \cos x \sin x}{\cos^2 x}$~~

Cancel $\frac{1+\sin x}{\cos x}$

Split $\frac{1}{\cos x} + \frac{\sin x}{\cos x}$

Reciprocal $\boxed{\sec x + \tan x}$

$\frac{\sec^2 x - \tan^2 x}{\sec x + \tan x} = \frac{1-\sin x}{\cos x}$

$\frac{1}{\sec x + \tan x}$ Pythag (Queen)

$\frac{1}{\cos x + \frac{\sin x}{\cos x}}$ Reciprocal a Reciprocal Identity

$\frac{\cos x}{1+\sin x}$ Combine

$\frac{\cos x (1-\sin x)}{1-\sin^2 x}$ Multiply by conjugate

$\frac{\cos x (1-\sin x)}{\cos^2 x}$ Pythag (King)

$\frac{1-\sin x}{\cos x}$ Cancel