Algebra 3-4

Name: *Key*

Unit 1 Summative 3 Review

Period:_____

IF.5 I can solve polynomial equations

5.3 Solving Polynomials (in text)
What are the real or imaginary solutions of each polynomial equation?

1._____

1. $(x - 1)(x^2 + 5x + 6) = 0$

$(x-1)(x+3)(x+2) = 0$

$\boxed{x=1}$ $\boxed{x=-3}$ $\boxed{x=-2}$

2. $(x + 2)(x^2 + 3x - 40) = 0$

$(x+2)(x+8)(x-5) = 0$

$\boxed{x=-2}$ $\boxed{x=-8}$ $\boxed{x=-5}$

2._____

3. $x^3 - 10x^2 + 16x = 0$

$x(x^2 - 10x + 16) = 0$

$x(x-8)(x-2) = 0$

$\boxed{x=0}$ $\boxed{x=8}$ $\boxed{x=2}$

4. $x^3 + 3x^2 - 54x = 0$

$x(x^2 + 3x - 54) = 0$

$x(x+9)(x-6) = 0$

$\boxed{x=0}$ $\boxed{x=-9}$ $\boxed{x=6}$

3._____

4._____

5. $125x^3 + 343 = 0$ SOAP

$(5x+7)(25x^2 - 35x + 49)$

$\boxed{x = \dfrac{-7}{5}}$ $25x^2 - 35x + 49 = 0$

Quadratic Formula

$x = \dfrac{35 \pm \sqrt{1225 - 4(25)(49)}}{2(25)}$

$x = \dfrac{35 \pm \sqrt{1225 - 4900}}{50}$

$x = \dfrac{35 \pm \sqrt{-3675}}{50}$

6. $x^3 = 216$ SOAP

$x^3 - 216 = 0$

$(x-6)(x^2 + 6x + 36) = 0$

QF

$\boxed{x=6}$

$x = \dfrac{-6 \pm \sqrt{36 - 4(36)}}{2}$

$x = \dfrac{-6 \pm \sqrt{-108}}{2}$

$x = \dfrac{-6 \pm \sqrt{36 \cdot 3}\,i}{2} \Rightarrow \dfrac{-6 \pm 6i\sqrt{3}}{2}$

$\boxed{x = -3 \pm 3i\sqrt{3}}$

$x = \dfrac{35 \pm 5i\sqrt{147}}{50}$

$\boxed{x = \dfrac{7 \pm i\sqrt{147}}{10}}$

5._____

6._____

7. $x^4 - 20x^2 + 64 = 0$

$(x^2 - 16)(x^2 - 4) = 0$

$(x-4)(x+4)(x-2)(x+2) = 0$

$x = 4 \quad x = -4 \quad x = 2 \quad x = -2$

8. $-2x^4 = -8x^3 - 42x^2$

$0 = 2x^4 - 8x^3 - 42x^2$

$0 = 2x^2(x^2 - 4x - 42)$

QF

$x = 0$

$x = \dfrac{4 \pm \sqrt{16 - 4(42)}}{2}$

$x = \dfrac{4 \pm \sqrt{184}}{2} = \dfrac{4 \pm 2\sqrt{46}}{2}$

7. _____

8. _____

5.4 (text) Polynomial Division

9. Is $(x - 2)$ a factor of $P(x) = x^3 + 2x^2 - 6x - 4$? If yes write the other factor.

9. _____

```
2 |  1   2   -6   -4
          2    8    4
     1   4    2    | 0
```

Yes $x^2 + 4x + 2$ is the other factor

10.   Divide $-3x^3 - 2x^2 - x - 2$ by $x - 2$ using any method.

10. _____

```
2 |  -3   -2   -1    -2
           -6  -16   -34
     -3   -8  -17   |-36
```

$-3x^2 - 8x - 17 + \dfrac{-36}{x-2}$

↑
Remainder

11.   Divide $x^3 + x^2 - x + 2$ by $x + 4$ using any method.

11. _____

```
-4 |  1    1   -1    2
          -4   12  -44
      1   -3   11  |-42
```

$x^2 - 3x + 11 + \dfrac{-42}{x+4}$

↑
Remainder

12.   Divide $4x^2 + 23x - 16$ by $(4x+3)$ using any method.

12. _____

```
-3/4 |  4   23   -16
            -3   -15
        4   20   |-31
```

$4x + 20 + \dfrac{-31}{(4x+3)}$

↑
Remainder

13. Use synthetic division to find $P(4)$ for $P(x) = x^4 + x^3 + 10x^2 + 9x - 6$. ✶ You won't need to do this

13. _____

```
4 |  1    1    10    9    -6
          4    20  120   516
     1    5    30  129  |510
```

510

14. Use the Rational Root Theorem to list all possible rational roots of the polynomial equation $x^3 - 6x^2 + 4x + 9 = 0$. Do not find the actual roots.

$$\frac{\pm 3, \pm 9, \pm 1}{\pm 1} \qquad \text{so} \quad \pm 3, \pm 9, \pm 1$$

Find all roots of the polynomial equation. (Use any method)

15. $2x^3 + 2x^2 - 19x + 20 = 0$

Possible zeros: $\pm 10, \pm 2, \pm 4, \pm 5, \pm 20, \pm 1$

$$\begin{array}{r|rrr} -4 & 2 & 2 & -19 & 20 \\ & & -8 & 24 & -20 \\ \hline & 2 & -6 & 5 & \boxed{0} \end{array} \qquad \boxed{X = -4}$$

$2x^2 - 6x + 5 \qquad QF \Rightarrow \dfrac{6 \pm \sqrt{36 - 4(2)(5)}}{4}$

$$\Rightarrow \frac{6 \pm \sqrt{-4}}{4} = \frac{6 \pm 2i}{4} = \boxed{\frac{3 \pm i}{2}}$$

16. $x^3 - 3x^2 - 5x + 15 = 0$

$x^2(x-3) - 5(x-3) = 0$

$(x^2 - 5)(x-3) = 0$

$x^2 = 5 \qquad \boxed{X = 3}$

$\boxed{X = \pm\sqrt{5}}$

17. $2x^4 - 5x^3 + 53x^2 - 125x + 75 = 0$ ※ Don't worry about this one!

$$\begin{array}{r|rrrrr} 1 & 2 & -5 & 53 & -125 & 75 \\ & & 1 & -4 & 49 & -75 \\ \hline & 2 & -4 & 49 & -75 & \boxed{0} \end{array} \qquad \boxed{X = 1}$$

$2x^3 - 4x^2 + 49x - 75 = 0$

$$\begin{array}{r|rrrr} 3/2 & 2 & -4 & 49 & -75 \\ & & 3 & -3/2 & +75 \\ \hline & 2 & -1 & \cancel{} & \boxed{0} \\ & & & 45.5 \end{array} \qquad \boxed{X = \frac{3}{2}}$$

$2x^2 - x + 45.5$

QF

$\dfrac{1 \pm \sqrt{1 - 4(2)(45.5)}}{4}$

$\dfrac{1 \pm \sqrt{-275}}{4} = \dfrac{1 \pm i\sqrt{275}}{4}$

Find all roots of the polynomial equation.

18. $x^4 - 6x^2 - 7x - 6 = 0$

[crossed out handwriting] Don't do

18._____

5.2 / 5.5 (in text) Writing a polynomial given its roots.

Write a polynomial in standard form with the given roots.

19._____

19.   2, 5, -7

(A+2) $(x-2)(x-5)(x+7)$

$(x^2-2x-5x+10)(x+7)$

$(x^2-7x+10)(x+7)$

$x^3-7x^2+10x+7x^2-49x+70$

$\boxed{x^3 -39x+70=y}$

20._____

20.   -1, 3±2i

$(x+1)(x-3)(x-2i)(x+2i)$

$(x^2-2x-3)(x^2+4)$

$x^4-2x^3-3x^2+4x^2-8x-12$

$\boxed{y=x^4-2x^3+x^2-8x-12}$

21.   -3, 4i

$(x+3)(x-4i)(x+4i)$

$(x+3)(x^2+16)$

21._____

$\boxed{y= x^3+3x^2+16x+48}$