Does $\sin(45° + 30°) = \sin 75°$?

Yes!

(Think $\sqrt{45+30} = \sqrt{75}$)

Does $\sin(45°) + \sin(30°) = \sin 75°$

No

(Think $\sqrt{45} + \sqrt{30} \neq \sqrt{75}$)

1. $\sin 22° \cos 13° + \cos 22° \sin 13°$ (↓u ↓v ↓u ↓v)

$\sin(u+v)$

$\sin(22+13) = \sin(35)$

2. $\cos 94° \cos 18° + \sin 94° \sin 18°$

$\cos(94-18)$ or $\cos(18-9$

$\cos(76)$ or $\cos(-76$

3. $\sin \dfrac{\pi}{3} \cos \dfrac{\pi}{7} - \sin \dfrac{\pi}{7} \cos \dfrac{\pi}{3}$ (↓u ↓v)

$\sin(\pi/3 - \pi/7)$

$\sin\left(\dfrac{7\pi}{21} - \dfrac{3\pi}{21}\right) = \boxed{\sin\left(\dfrac{4\pi}{21}\right)}$

4. $\dfrac{\tan \dfrac{\pi}{2} - \tan \dfrac{\pi}{3}}{1 + \tan \dfrac{\pi}{2} \tan \dfrac{\pi}{3}}$ $\tan(\pi/6)$

Practice: Try using special right triangle angles

What special angles can you use to make:

$15°$ $45° - 30°$

      $330° - 315°$

      etc.

$\dfrac{11\pi}{12} = \dfrac{2\pi}{3} + \dfrac{\pi}{4}$

$75° \quad 30° + 45°$

$\dfrac{\pi}{12} = \boxed{\dfrac{\pi}{3} - \dfrac{\pi}{4}}$

5. $\cos 75°$

$\cos(30+45)$

$\cos 30 \cos 45 - \sin 30 \sin 45$

$\dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} = \boxed{\dfrac{\sqrt{6} - \sqrt{2}}{4}}$

Prove the identity.

7. $\cos\left(x - \dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}(\cos x + \sin x)$

6. $\sin \dfrac{7\pi}{12} = \sin\left(\dfrac{\pi}{3} + \dfrac{\pi}{4}\right)$

$\Rightarrow \sin \dfrac{\pi}{3} \cos \dfrac{\pi}{4} + \sin \dfrac{\pi}{4} \cos \dfrac{\pi}{3}$

$\dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2}$

$\boxed{\dfrac{\sqrt{6} + \sqrt{2}}{4}}$

Sum and Difference Formulas
Homework

Part 1: Find the exact value of each expression or angle below.

1. $\sin(135° - 30°)$

$\sin(135)\cos(30) - \cos(135)\sin(30)$

$\dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2}$

$$\boxed{\dfrac{\sqrt{6} - \sqrt{2}}{4}}$$

2. $\sin 135° - \sin 30°$

$\dfrac{\sqrt{2}}{2} - \dfrac{1}{2}$

$$\boxed{\dfrac{\sqrt{2} - 1}{2}}$$

3. $\sin 105°$

$\sin(150 - 45)$

$\sin 150 \cos 45 - \sin 45 \cos 150$

$\dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2}$

$$\boxed{\dfrac{\sqrt{2} - \sqrt{6}}{4}}$$

4. $\cos 105°$

$\cos(150 - 45)$

$\cos 150 \cos 45 + \sin 150 \sin 45$

$\dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} + \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2}$

$$\boxed{\dfrac{\sqrt{6} + \sqrt{2}}{4}}$$

5. $\sin \dfrac{13\pi}{12}$

$\dfrac{9\pi}{12} + \dfrac{4\pi}{12}$

$\sin\left(\dfrac{3\pi}{4} + \dfrac{\pi}{3}\right) =$

$\sin \dfrac{3\pi}{4} \cos \dfrac{\pi}{3} + \cos \dfrac{3\pi}{4} \sin \dfrac{\pi}{3}$

$\dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} + \dfrac{-\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} = \boxed{\dfrac{\sqrt{2} - \sqrt{6}}{4}}$

6. $\sin \dfrac{7\pi}{12}$

$\sin \dfrac{\pi}{3} \cos \dfrac{\pi}{4} + \sin \dfrac{\pi}{4} \cos \dfrac{\pi}{3}$

$\dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2}$

$\dfrac{\sqrt{6}}{4} + \dfrac{\sqrt{2}}{4}$

$$\boxed{\dfrac{\sqrt{6} + \sqrt{2}}{4}}$$

7. $\cos \dfrac{7\pi}{12}$

$\cos \dfrac{\pi}{3} \cos \dfrac{\pi}{4} - \sin \dfrac{\pi}{3} \sin \dfrac{\pi}{4}$

$\dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2}$

$$\boxed{\dfrac{\sqrt{2} - \sqrt{6}}{4}}$$

8. $\sin 285°$

$\sin(315 - 30)$

$\sin 315 \cos 30 - \sin 30 \cos 315$

$-\dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2}$

$$\boxed{\dfrac{\sqrt{6} - \sqrt{2}}{4}}$$

9. $\cos \dfrac{19\pi}{12}$

$\dfrac{9\pi}{12} + \dfrac{10\pi}{12}$

$\cos\left(\dfrac{3\pi}{4} + \dfrac{5\pi}{6}\right)$

$\cos \dfrac{3\pi}{4} \cos \dfrac{5\pi}{6} - \sin \dfrac{3\pi}{4} \sin \dfrac{5\pi}{6}$

$\left(-\dfrac{\sqrt{2}}{2}\right)\left(-\dfrac{\sqrt{3}}{2}\right) - \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{1}{2}\right)$

$$\boxed{\dfrac{\sqrt{6} - \sqrt{2}}{4}}$$

Part 2: Write the expression as the sine or cosine of an angle.

1. $\cos 25° \cos 15° - \sin 25° \sin 15°$

$\cos(25 + 15) = \boxed{\cos 40}$

2. $\sin \dfrac{\pi}{6} \cos \dfrac{\pi}{3} - \cos \dfrac{\pi}{6} \sin \dfrac{\pi}{3}$

$\sin\left(\dfrac{\pi}{6} - \dfrac{\pi}{3}\right)$

$$\boxed{\sin\left(-\dfrac{\pi}{6}\right)}$$

Part 3: Find the exact value of each angle below.

1. $\tan 105°$

$\tan(150 - 45) = \dfrac{\tan(150) - \tan(45)}{1 + \tan(150)\tan(45)} = \dfrac{-\dfrac{\sqrt{3}}{3} - 1}{1 + \left(-\dfrac{\sqrt{3}}{3}\right)(1)}$

2. $\tan 225° = 1$

$= \dfrac{-\sqrt{3} - 3}{3} \cdot \dfrac{3}{3 - \sqrt{3}} = \boxed{\dfrac{-\sqrt{3} - 3}{3 - \sqrt{3}}}$

3. $\tan -\dfrac{7\pi}{12} = \tan\left(-\dfrac{\pi}{3} - \dfrac{\pi}{4}\right)$

$\dfrac{\tan\left(-\dfrac{\pi}{3}\right) - \tan\left(-\dfrac{\pi}{4}\right)}{1 + \tan\left(-\dfrac{\pi}{3}\right)\left(\dfrac{\pi}{4}\right)} = \boxed{\dfrac{1 - \sqrt{3}}{1 + \sqrt{3}}}$