Rewrite w/ cosines

(1) $\cos^2 x - (1-\cos^2 x) + 3\cos x - 1 = 0$

$2\cos^2 x + 3\cos x - 2 = 0$

Factor (Think $2x^2 + 3x - 2 = 0$)

$(2\cos x - 1)(\cos x + 2) = 0$

So $2\cos x - 1 = 0$ or $\cos x = -2$

$\cos x = \frac{1}{2}$ $\quad\quad \emptyset$

$\Rightarrow \boxed{x = \pi/3, \frac{5\pi}{3}}$

(2) $2\sin x\cos x = 2\sin x$ Simplify divide by 2

$\sin x \cos x = \sin x$ divide by $\sin(x)$

$\cos x = 1 \quad \Rightarrow \boxed{x = 0, 2\pi}$

(3) $\tan^2 x - \sec^2 x = 1$

$\tan^2 x - (\tan^2 x + 1) = 1$ Queen Pythag.

$-1 = 1$ $\boxed{\text{NO SOLUTION}}$

(4) $3\sin x\cos^2 x + \sin^3 x = \sin x$ Factor GCF

$\sin x(3\cos^2 x + \sin^2 x) = \sin x$ Cannot use KING b/c 3!

$\sin x(3(1-\sin^2 x) + \sin^2 x) = \sin x$ divide by $\sin x$

$3(1-\sin^2 x) + \sin^2 x = 1$

$3 - 3\sin^2 x + \sin^2 x = 1$

$-2\sin^2 x = -2$

$\sin^2 x = 1 \Rightarrow \sin x = 1$ OR $\sin x = -1$ So $x = \frac{\pi}{2}, \frac{3\pi}{2}$

(5) $3\sin^3 x = \sin x\cos^2 x$

$3\sin^2 x = \cos^2 x$

$3\sin^2 x - \cos^2 x = 0$

$3\sin^2 x - (1 - \sin^2 x) = 0$

$4\sin^2 x - 1 = 0$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

) $3\cos^2 x - 5\cos x = 1 + 3\sin^2 x$

$\cancel{\cos x(3\cos x - 5)} = 1 + 3(1 - \cos^2 x)$

$3\cos^2 x - 5\cos x = 1 + 3 - 3\cos^2 x$

$6\cos^2 x - 5\cos x - 4 = 0$ Factor

(Think $6x^2 - 5x - 4 = 0$)

$(3\cos x - 4)(2\cos x + 1) = 0$

So $3\cos x - 4 = 0$ OR $2\cos x + 1 = 0$

$\cos x = \frac{4}{3}$ $\quad$ $\cos x = -1/2$

$\varnothing$ $\quad$ $\boxed{x = \frac{2\pi}{3}, 4\pi/3}$

7) $\cos^2 x - \sin^2 x = 2\cos x - 1$

$\cos^2 x - (1 - \cos^2 x) = 2\cos x - 1$

$2\cos^2 x - 1 = 2\cos x - 1$

$2\cos^2 x - 2\cos x = 0$

$2\cos x(\cos x - 1) = 0$

So either $2\cos x = 0$ OR $\cos x - 1 = 0$

$\boxed{x = \pi/2, 3\pi/2, 0, 2\pi}$

8) $1 + \cos x = 2\sin^2 x$

$1 + \cos x = 2(1 - \cos^2 x)$ distribute & move to left

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

So either $2\cos x - 1 = 0$ OR $\cos x + 1 = 0$

$\cos x = 1/2$ $\quad$ $\cos x = -1$

$\boxed{x = \pi/3, 5\pi/3 \quad x = \pi}$

9) $\sin^2 x - 3\cos x - 1 = 0$

$(1-\cos^2 x) - 3\cos x - 1 = 0$

$-\cos^2 x - 3\cos x = 0$

$-\cos x(\cos x + 3) = 0$

So either $-\cos x = 0$ or $\cos x + 3 = 0$

$\boxed{x = \pi/2, 3\pi/2}$ $\qquad \emptyset$

10) $\sqrt{3}\csc^2 x + 2\csc x = 0$

$\csc x(\sqrt{3}\csc x + 2) = 0$

So either $\csc x = 0$ or $\sqrt{3}\csc x + 2 = 0$

$\Rightarrow \frac{1}{\sin x} = 0$ $\qquad \csc x = -\frac{2}{\sqrt{3}}$

$\boxed{\text{NEVER}}$ $\qquad (\text{or } \sin x = -\sqrt{3}/2)$

So $\boxed{x = \frac{4\pi}{3}, \frac{5\pi}{3}}$

WORKSHEET 2

(1) $\csc^2 x - 2\cot x = 0$

$(1 + \cot^2 x) - 2\cot x = 0$

$\cot^2 x - 2\cot x + 1 = 0$

$(\cot x - 1)(\cot x - 1) = 0$

So $\cot x - 1 = 0$ or $\cot x = 1$

$\Rightarrow \boxed{x = \pi/4, 5\pi/4}$

(2) $2 = \sec x + \sec^2 x$

$0 = \sec^2 x + \sec x - 2$

$0 = (\sec x + 2)(\sec x - 1)$

So $\sec x + 2 = 0$ OR $\sec x - 1 = 0$

$\sec x = -2$ $\qquad \sec x = 1$

when $\sin x = -1/2$ $\qquad$ when $\sin x = 1$

So $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$