Name______________

Rational Root Theorem

Any possible "root" (solution, zero) can be written as

$$\frac{p}{q} = \frac{factors\ of\ constant\ term}{factors\ of\ the\ lead\ coefficient}$$

Example:

$$3x^4 + 4x^3 - 2x + 7 = 0$$

Possible roots: $\frac{\pm 7, \pm 1}{\pm 3, \pm 1}$ or in other words: $\frac{7}{3}, \frac{-7}{3}, \frac{1}{3}, \frac{-1}{3}, -1, 1$, 7, -7

If the possible roots are

$$\frac{\pm 3, \pm 5, \pm 15, \pm 1}{\pm 4, \pm 2, \pm 1}$$

List all possible roots individually:

$\pm\frac{3}{4}$, $\pm\frac{3}{2}$, ± 3, $\pm\frac{5}{4}$, $\pm\frac{5}{2}$, ± 5,

$\pm\frac{15}{4}$, $\pm\frac{15}{2}$, ± 15, $\pm\frac{1}{4}$, $\pm\frac{1}{2}$, ± 1

Homework. Find all possible roots of the following:

1) $f(x) = 3x^2 + 2x - 1$

$\frac{p}{q} = \frac{-1, 1}{-1, 1, -3, 3}$

$\Rightarrow \pm 1, \pm \frac{1}{3}$

2) $f(x) = x^6 - 64$

$\frac{\pm 8, \pm 2, \pm 32, \pm 64, \pm 1}{\pm 1}$

So: $\pm 8, \pm 2, \pm 32, \pm 64, \pm 1$

3) $f(x) = x^2 + 8x + 10$

$\boxed{\pm 5, \pm 2, \pm 10, \pm 1}$

4) $f(x) = 5x^3 - 2x^2 + 20x - 8$

$\frac{\pm 4, \pm 2, \pm 8, \pm 1}{\pm 5, \pm 1}$

$\Rightarrow \boxed{\pm \frac{4}{5}, \pm 4, \pm \frac{2}{5}, \pm 2, \pm \frac{8}{5}, \pm 8, \pm 1, \pm \frac{1}{5}}$

5) $f(x) = 4x^5 - 2x^4 + 30x^3 - 15x^2 + 50x - 25$

$\frac{\pm 5, \pm 25, \pm 1}{\pm 2, \pm 4, \pm 1}$

$\Rightarrow \pm \frac{5}{2}, \pm \frac{5}{4}, \pm 5, \pm \frac{25}{2}, \pm \frac{25}{4}, \pm 25, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 1$

6) $f(x) = 5x^4 + 32x^2 - 21$

$\frac{\pm 7, \pm 3, \pm 21, \pm 1}{\pm 5, \pm 1}$

$\boxed{\Rightarrow \pm \frac{7}{5}, \pm 7, \pm \frac{3}{5}, \pm 3, \pm \frac{21}{5}, \pm 21, \pm \frac{1}{5}, \pm 1}$

7) $f(x) = x^3 - 27$

$\boxed{\frac{\pm 27, \pm 1, \pm 9, \pm 3}{\pm 1}}$

8) $f(x) = 2x^4 - 9x^2 + 7$

$\frac{\pm 7, \pm 1}{\pm 2, \pm 1}$

$\Rightarrow \boxed{\pm \frac{7}{2}, \pm 7, \pm \frac{1}{2}, \pm 1}$

Which possible root(s) do all of the polynomials have in common?

± 1