

**U8LT3 I can solve and prove multiple angle identities.**Show Your Work. Find an exact value using angle sum and difference formulas.

1..  $\sin 105^\circ$

1. \_\_\_\_\_

$$\sin(45+60) = \sin 45 \cos 60 + \cos 45 \sin 60$$

$$\frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$$

2.  $\cos \frac{13\pi}{12}$

2. \_\_\_\_\_

$$\cos\left(\frac{5\pi}{6} + \frac{\pi}{4}\right) = \cos\left(\frac{5\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{5\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{-\sqrt{6} - \sqrt{2}}{4}}$$

Write the express as the sine, cosine, or tangent of a single angle.

3.  $\cos \frac{\pi}{5} \cos \frac{\pi}{7} + \sin \frac{\pi}{5} \sin \frac{\pi}{7}$

3. \_\_\_\_\_

$$\cos\left(\frac{\pi}{5} - \frac{\pi}{7}\right) = \cos\left(\frac{7\pi - 5\pi}{35}\right) = \boxed{\cos\left(\frac{2\pi}{35}\right)}$$

4.  $\frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$

4. \_\_\_\_\_

$$= \tan(45 - 30) = \boxed{\tan 15}$$

Prove the identity.

5.  $\sin\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\sin x - \cos x)$

~~$\sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4}$~~

$$\sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4}$$

$$\sin x \frac{\sqrt{2}}{2} - \cos x \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2}(\sin x - \cos x) \quad \square$$

6.  $\cos 4u = \cos^2 2u - \sin^2 2u$

$$\cos 4u = \cos(2u+2u)$$

$$= \cos 2u \cos 2u - \sin 2u \sin 2u$$

$$= \boxed{\cos^2 2u - \sin^2 2u} \quad \square$$

Find all solutions to the equation in the interval  $[0, 2\pi)$ .

7.  $\sin 2x = -\sin x$

7. \_\_\_\_\_

~~$2\sin 2x$~~

$2\sin x \cos x = -\sin x$

$2\sin x \cos x + \sin x = 0$

$\sin x (2\cos x + 1) = 0$

$\sin x = 0$

$2\cos x + 1 = 0$

$x = 0, \pi$

$\cos x = -1/2$

$x = +\frac{2\pi}{3}, \frac{4\pi}{3}$

Find the exact value by using half-angle identity.

8.  $\sin 22.5^\circ$

8. \_\_\_\_\_

$\Rightarrow \sin\left(\frac{45}{2}\right) = \pm \sqrt{\frac{1 - \cos 45}{2}} = \pm \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$  or  $\pm \sqrt{\frac{2 - \sqrt{2}}{4}}$

9.  $\tan \frac{7\pi}{8} = \tan\left(\frac{14\pi/8}{2}\right) = \tan\left(\frac{7\pi/4}{2}\right)$

9. \_\_\_\_\_

$\Rightarrow \frac{1 - \cos(7\pi/4)}{\sin(7\pi/4)} = \frac{1 - \frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{2}}{\sqrt{2}} \cdot \frac{2}{-2} = -\frac{2 - \sqrt{2}}{\sqrt{2}}$

$= -\frac{2 - \sqrt{2}}{\sqrt{2}}$  or  $\frac{2\sqrt{2} - 2}{2} = \frac{-\sqrt{2} + 1}{1}$

U8LT3



OR  $\frac{\sin(7\pi/4)}{1 + \cos(7\pi/4)} = \frac{-\frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}} = \frac{-\sqrt{2}}{2} \cdot \frac{2}{2 + \sqrt{2}} = \frac{-\sqrt{2}}{2 + \sqrt{2}}$