U8LT1 Proving Trig ID

1. $(1+\cos x)(1-\cos x)=\sin^2 x$

$1+\cos x-\cos x-\cos^2 x=\sin^2 x$ Distribute

$1-\cos^2 x=\sin^2 x$ Cancel like terms

$\sin^2 x=\sin^2 x$ Pythag. Identity (KING)

$\square$

2. $\csc x+\tan x=\csc x\sec x$

$\frac{1}{\sin x}+\frac{\sin x}{\cos x}$

$\frac{\cos x+\sin^2 x}{\sin x\cos x}$

$\frac{\cos x+1-\cos^2 x}{\sin x\cos x}\neq \csc x\sec x$

$\boxed{\text{No Solution}}$

3. $\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}$

$\boxed{\text{LHS}}$

$\frac{(1-\cos x)\sin x}{(1-\cos x)(1+\cos x)}$ Mult. by conjugate

$\frac{\sin x-\sin x\cos x}{1-\cos^2 x}$ Distribute

$\frac{\sin(x)(1-\cos x)}{\sin^2 x}$ Pythag. Identity KING

$\frac{1-\cos x}{\sin x}$ Cancel Sine on top/bottom

$\square$

4. $\frac{1}{\sin x\cos x}-\frac{\cos x}{\sin x}=\tan x$

$\frac{1}{\sin x\cos x}-\frac{\cos^2 x}{\sin x\cos x}$ Common denominators

$\frac{1-\cos^2 x}{\sin x\cos x}$ Subtract Fractions

$\frac{\sin^2 x}{\sin x\cos x}$ Pythag identity (KING)

$\frac{\sin x}{\cos x}$ Cancel Sine

$\tan x$ Ratio Identity

$\square$

5. $\frac{1-\sin x}{\cos x}=\frac{\cos x}{1+\sin x}$

$\boxed{\text{RHS}}$

$\frac{\cos x}{1+\sin x}\frac{(1-\sin x)}{(1-\sin x)}$ Mult. by conjugate

$\frac{\cos x(1-\sin x)}{1-\sin^2 x}$ Distribute

$\frac{\cos x(1-\sin x)}{\cos^2 x}=\frac{1-\sin x}{\cos x}$ Cancel

$\square$

6. $\sin^2 x+\sin x\cos x\cot x=1$

~~$\sin x(\sin x+\cos x)$~~ ~~$\cos x$~~

$\sin^2 x+\sin x\cos x\frac{\cos x}{\sin x}$ Ratio Identity

$\sin^2 x+\cos^2 x$ Simplify

1 Pythag. Identity (KING)

$\square$

7. $\tan x+\cot x=\sec x\csc x$

$\boxed{\text{LHS}}$

$\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}$ Ratio Identities

$\frac{\sin^2 x+\cos^2 x}{\cos x\sin x}$ Common denom to add fractions

$\frac{1}{\cos x\sin x}$ Pythag. Identity (KING!)

$\frac{1}{\cos x}\cdot\frac{1}{\sin x}=\sec x\csc x$

$\square$

8. $\frac{\cos x}{1+\sin x}+\frac{\cos x}{1-\sin x}=2\sec x$

$\frac{\cos x(1-\sin x)+\cos x(1+\sin x)}{(1+\sin x)(1-\sin x)}$ Common denom to add

$\frac{\cos x-\cos x\sin x+\cos x+\cos x\sin x}{1-\sin^2 x}$ Distribute

$\frac{\cos x+\cos x}{\cos^2 x}$ Combine like terms

$\frac{2\cos x}{\cos^2 x}=2\sec x$ Simplify

Reciprocal Identity

$\square$

8. $\csc x\cos^2 x + \sin x = \cos x$

$\frac{1}{\sin x}\cos^2 x + \sin x$

$\frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}$

$\boxed{\text{Not possible}}$

$\frac{1}{\sin x} \neq \cos x$

9. $\sec^2 x + \csc^2 x = \sec^2 x\,\csc^2 x$

$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$ Reciprocal Identities

$\frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$ Common denominators

Pythag (KING) $\frac{1}{\cos^2 x\sin^2 x} = \frac{1}{\cos^2 x}\cdot\frac{1}{\sin^2 x} = \sec^2 x\csc^2 x$

Split Fractions

Recip. Identity □

10. $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = 1$

$\cos^2 x + \sin^2 x = 1$

Reciprocal Identities

□

Pythag. Identity (KING)

Hard 11. $\frac{1}{1-\sin x} = \sec^2 x + \sec x\tan x$

$\boxed{\text{RHS}}$

$\frac{\cancel{(1+\sin x)}}{\cancel{(1+\sin x)(1-\sin x)}}$

$\cancel{1+\sin x}$

$\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$ Recip & Ratio Identity

$\frac{1+\sin x}{(\cos^2 x)}$ Add Fractions

Pythag. Identity (KING)

$\frac{1+\sin x}{(1-\sin^2 x)} \Rightarrow \frac{\cancel{1+\sin x}}{(\cancel{1+\sin x})(1-\sin x)} = \boxed{\frac{1}{1-\sin x}}$

Factor: Diff of Perfect Squares

□ Simplify

12. $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$

$\frac{\sin^2 x}{1} + \frac{\cos^2 x}{1}$ Reciprocal Identities

$\boxed{1}$

□ Pythag. Identity (KING)

13. $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = 2\csc x$

$\frac{\sin^2 x}{\sin x(1+\cos x)} + \frac{(1+\cos x)(1+\cos x)}{\sin x(1+\cos x)}$

$\frac{\sin^2 x + 1 + \cos^2 x + 2\cos x}{\sin x(1+\cos x)}$

$\frac{2+2\cos x}{\sin x(1+\cos x)}$

$\frac{2(1+\cancel{\cos x})}{\sin x(1+\cancel{\cos x})} = \frac{2}{\sin x} = 2\csc x$ □

14. $\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

$\frac{(1-\cos x)\sin x}{(1-\cos x)(1+\cos x)}$ Multiply by Conjugate

$\frac{\sin x(1-\cos x)}{1-\cos^2 x}$ Distribute

$\frac{\sin x(1-\cos x)}{\sin^2 x}$ Pythag. Identity (KING)

$\frac{1-\cos x}{\sin x}$ Cancel Sine.

15. $\sec x + \tan x = \frac{1}{\sec x - \tan x}$

$\frac{1}{(\sec x-\tan x)}\frac{(\sec x+\tan x)}{(\sec x+\tan x)}$ Mult. by Conjugate

$\frac{\sec x+\tan x}{\sec^2 x-\tan^2 x}$ Distribute.

$\sec x+\tan x$ Pythag. identity (QUEEN)