Algebra 3-4 Name: Key

U3IF3 Review: Solving Exponential Growth and Decay Problems Period:

$A(t) = Pe^{rt}$ $y = a_0(\frac{1}{2})^{\frac{t}{h}}$ $A(t) = a_o(1 \pm r)^t$

r = rate as a decimal $a_0 = initial\ amount$ $e = euler's\ constant\ on\ your\ calculator$

t = time h = half life time P = principal

1. Write a formula that represents the average growth of the population of a city with a rate of 7.5% per year. Let x represent the number of years, y represent the most recent total population of the city, and A is the city's population now. What is the expected population in 10 years if the city's population now is 22,750 people?

r=0.075

$y = A(1+.075)^x$

X=10
A=22,750

$y = 22{,}750(1+.075)^{10}$

$y = 5667.83$

⇒ 5668 people

2. Suppose the value of a computer depreciates (less) at a rate of 25% a year. Write a formula to represent this model. Determine the value of a laptop computer two years after it has been purchased for $3,750.

r=.25
X=2
A=3,750

$y = A(1-.25)^x$

$y = 3750(1-.25)^2$ = $2109.38

3. In 2012 New Zealand had a population of about 4,444,444 million people with an average growth rate of 0.83%. In what year will New Zealand break the 7 million person barrier? A Y

r=.0083

$y = 4{,}444{,}444(1+.0083)^x$

$7{,}000{,}000 = 4{,}444{,}444(1.0083)^x$

*Plug in 7 million → Solve for x

$1.575 = 1.0083^x$

$\log_{1.0083} 1.575 = x$

X=54.96 years so 2067

4. A researcher estimates that the initial population of honeybees in a colony is 500. They are increasing at a rate of 14% per week. What is the expected population in 22 weeks?

A r=.14 X

$y = A(1+r)^x$

$y = 500(1+.14)^{22} = 8930.519 \approx$ 8931 bees

5. Radioactive gold 198 (^{198}Au), used in imaging the structure of the liver, has a half-life of 2.67 days. If the initial amount is 50 milligrams of the isotope, how many milligrams *(rounded to the nearest tenth)* will be left over after:

(a) ½ day
(b) 1 week

(a) $y = 50(\frac{1}{2})^{0.5/2.67}$ h=2.67 t=0.5 days

y=43.91 milligrams

(b) $y = 50(\frac{1}{2})^{7/2.67}$ h=2.67 t=7 days

y=8.123 milligrams

6. If a farmer uses 25 pounds of insecticide, assuming its half-life is 12 years, how many pounds *(rounded to the nearest tenth)* will still be active after:

(a) 5 years
(b) 20 years

(a) $t=5$ $y = 25\left(\frac{1}{2}\right)^{5/12} = 18.73$ pounds

(b) $t=20$ $y = 25\left(\frac{1}{2}\right)^{20/12} = 7.875$ pounds

7. Suppose a certain type of bacteria reproduces according to the model $B = 100\,e^{0.271t}$, where t is the time in hours.

(a) At what percentage rate does this type of bacteria reproduce? What is the rate? $r = .271$ So percent-wise 27%

(b) What was the initial number of bacteria? 100

(c) Find the number of bacteria *(rounded to the nearest whole number)* after:

(i) 5 hours $B = 100e^{0.271(5)} = 1938.38$

24 hrs (ii) 1 day $B = 100e^{0.271(24)} = 66780.75$

72 hrs (iii) 3 days $B = 100e^{0.271(72)} = 2.978 * 10^{10}$

9. A promissory note will pay $30,000 at maturity 10 years from now. How much should you be willing to pay for the note now if the note gains value at a rate of 9% compounded continuously?

$30,000 = Pe^{(.09)(10)}$

Solve for P

$30,000 = P(2.4596)$

$P = \$12197.09$

10. Suppose Jorge deposits $1,500 in a savings account that earns 6.75% interest compounded continuously. He plans to withdraw the money in 6 years to make a $2,500 down payment on a car. Will there be enough funds in his account in 6 years to meet his goal? Explain your answer.

$y = Pe^{rt} = 1500e^{.0675 \cdot 6} = \$2248.95$ Not quite! only

11. Tim and Kerry are saving for their daughter's college education. If they deposit $12,000 in an account bearing 6.4% interest compounded continuously, how much will be in the account when she goes to college in 12 years?

$y = Pe^{rt}$

$y = 12000e^{.064 \cdot 12} = \$25865.41$

12. Noah invests $3500 in an account yielding 3.75% interest compounded monthly. What will the account balance be in 2 years assuming he makes no further deposits or withdrawals?

tricky! 12 times per year

24 months

$y = A(1+r)^t$

$y = 3500(1+.0375)^{24}$

$= \$8468.03$