

Name: \_\_\_\_\_

## Simplifying Trig Expressions

Simplify the following trig expressions completely:

$$1. \frac{\tan^2 x + 1}{1 + \cot^2 x} = \frac{\sec^2 x}{\csc^2 x} = \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}$$

$$\Rightarrow \frac{\sin^2 x}{\cos^2 x} = \boxed{\tan^2 x}$$

$$2. \frac{1}{\sec x - \tan x} - \frac{1}{\sec x + \tan x}$$

$$\frac{\sec x + \tan x - (\sec x - \tan x)}{\sec^2 x - \tan^2 x} = \frac{2 \tan x}{1}$$

$$\Rightarrow \boxed{2 \tan x}$$

$$3. \sec x \tan x \cos x$$

$$\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} = \boxed{\frac{\sin x}{\cos x}}$$

$$4. \sin^2 x \cot x \csc x$$

$$\sin^2 x \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = \boxed{\cos x}$$

$$5. \frac{1 - \cos^2 t}{\sin^2 t} = \frac{\sin^2 t}{\sin^2 t} = \boxed{1}$$

$$6. \frac{\tan^2 x}{1 - \sec^2 x} = \frac{\tan^2 x}{-\tan^2 x} = \boxed{-1}$$

$$7. \tan^2 x (\csc^2 x - 1)$$

$$\tan^2 x (\cot^2 x) = \boxed{1}$$

$$8. \frac{\cos^2 x}{1 - \cos^2 x} = \frac{\cos^2 x}{\sin^2 x} = \boxed{\cot^2 x}$$

$$9. \frac{\sec^2 x - 1}{\tan x} = \frac{\tan^2 x}{\tan x} = \boxed{\tan x}$$

$$10. \frac{\cos^2 x - 1}{\sin^2 x - 1} = \frac{\sin^2 x}{\cos^2 x} = \boxed{\tan^2 x}$$

$$11. \cos x (\sec x - \cos x)$$

$$\cos x \left( \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} \right)$$

$$\cos x \left( \frac{1 - \cos^2 x}{\cos x} \right) = \frac{1 - \cos^2 x}{\sin^2 x}$$

$$= \boxed{\sin^2 x}$$

$$12. \cot x (\tan x + \cot x)$$

$$\cot x \left( \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)$$

$$\cot$$

$$13. \frac{\tan x + \cot x}{\cot x} = \cancel{\frac{\sin x}{\cos x}} = \frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x}}$$

$$\Rightarrow \frac{\sin x}{\cos x} \left( \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} \right) \Rightarrow \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$

$$14. \frac{\tan x}{\tan x + \cot x} = \frac{\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} \Rightarrow \frac{\frac{\sin x}{\cos x}}{\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}}$$

$$\frac{\sin x \cdot \cos x \sin x}{\cos x} = \boxed{\sin^2 x}$$

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15.  $\sec x \cot x - \cot x \cos x$

$$\frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} - \frac{\cos x}{\sin x} \cdot \cos x$$

$$\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x} \Rightarrow \frac{\sin^2 x}{\sin x} = \boxed{\sin x}$$

16.  $\sin x \tan x - \csc x \tan x$

$$\frac{\sin^2 x}{\cos x} - \frac{1}{\cos x} = \frac{\cos^2 x}{\cos x}$$

$$\boxed{\cos x}$$

17.  $\frac{\cot^2 x \cos^2 x}{\cot^2 x - \cos^2 x}$

$$\frac{\cot^2 x \cos^2 x}{\frac{\cos^2 x}{\sin^2 x} - \frac{\cos^2 x \sin^2 x}{\sin^2 x}} = \frac{\frac{\cos^2 x \cos^2 x}{\sin^2 x}}{\frac{\cos^2(1-\sin^2 x)}{\sin^2 x}}$$

$$\boxed{\cos^2 x}$$

18.  $\frac{\sin^2 x - \tan^2 x}{\tan^2 x \sin^2 x}$

$$\frac{\sin^2 x - \frac{\sin^2 x}{\cos^2 x}}{\frac{\sin^4 x}{\cos^2 x}}$$

$$\frac{\sin^2 x \cos^2 x - \sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^4 x}$$

$$\frac{\sin^2 x (\cos^2 x - 1)}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^4 x} = \boxed{1}$$

19.  $\frac{(\sin x + \tan x)^2 + \cos^2 x - \sec^2 x}{\tan x}$

$$\sin^2 x + 2\sin x \tan x + \tan^2 x + \cos^2 x - (\tan^2 x + 1)$$

$$\frac{2\sin x \tan x}{\tan x} = \boxed{2\sin x}$$

20.  $\frac{2 \sin x \cos x + (\sin x - \cos x)^2}{\sec x}$

$$\text{Top: } 2\sin x \cos x + \sin^2 x - 2\sin x \cos x + \cos^2 x$$

$$\frac{\sin^2 x + \cos^2 x}{\sec x} = \frac{1}{\sec x} = \boxed{\cos x}$$

21.  $\frac{\sin x(1+\sin x)}{1-\cos^2 x} - 1$

$$\frac{\sin x(1+\sin x) - (1-\cos^2 x)}{1-\cos^2 x}$$

$$\frac{\sin x + \sin^2 x - 1 + \cos^2 x}{\sin^2 x}$$

$$= \frac{\sin x}{\sin^2 x} = \frac{1}{\sin x}$$

$$\boxed{\csc x}$$

22.

$$\frac{1-\tan^2 x}{1+\tan^2 x} + 1$$

$$\frac{1-\tan^2 x}{\sec^2 x} + \frac{\sec^2 x}{\sec^2 x} \Rightarrow \frac{1-(\sec^2 x - 1) + \sec^2 x}{\sec^2 x}$$

$$\frac{\tan x - \tan x \sin^2 x}{2 \sin x \cos x}$$

$$\frac{\tan x (1 - \sin^2 x)}{2 \sin x \cos x} = \frac{\tan x \cos^2 x}{2 \sin x \cos x} = \boxed{\frac{1}{2}}$$

23.

$$\frac{\sec x - \cos x}{3 \tan x \sin x} = \frac{\frac{1}{\cos x} - \cos x}{3 \frac{\sin^2 x}{\cos x}} = \frac{\frac{1-\cos^2 x}{\cos x}}{3 \frac{\sin^2 x}{\cos x}}$$

$$\Rightarrow \boxed{\frac{1}{3}}$$

24.

$$\frac{\sin^3 x + \cos^3 x}{1 - \sin x \cos x}$$

$$\frac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{1 - \sin x \cos x}$$

26.

$$\frac{\cos^3 x + \sin^3 x}{(\cos x + \sin x)^2}$$

$$\frac{(\cos x + \sin x)(\cos^2 x - \sin x \cos x + \sin^2 x)}{(\cos x + \sin x)(\sin x + \cos x)}$$

$$\boxed{\frac{1 - \sin x \cos x}{\sin x \cos x}}$$

27.

$$\frac{1 + \tan^2 x + \sec^2 x \cot^2 x}{\csc^2 x + \cot^2 x \csc^2 x}$$

$$\frac{\sec^2 x + \sec^2 x \cot^2 x}{\csc^2 x (1 + \cot^2 x)} = \frac{\sec^2 x (1 + \cot^2 x)}{\csc^2 x (1 + \cot^2 x)}$$

$$\frac{\sin^2 x}{\cos^2 x} \Rightarrow \boxed{\tan^2 x}$$

28.

$$\frac{\cot x \sec^2 x - \cot x}{\sin x \tan x + \cos x}$$

$$\frac{\cot x (\sec^2 x - 1)}{\frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}} \Rightarrow \frac{\cot x \tan^2 x}{\frac{1}{\cos x}}$$

$$\Rightarrow \frac{\cos x \cot x \tan^2 x}{\frac{\cos^2 x}{\sin x} \cdot \frac{\sin^2 x}{\cos^2 x}} = \boxed{\sin x}$$