# Optimization HW

1. Find two positive numbers such that the product is 48 and the sum of the first plus three times thte second is a minimum.

$a \cdot b = 48$ ← Condition

→ minimize $a + 3b$

① Rewrite condition in terms of b

$b = \frac{48}{a}$

② Rewrite function to minimize in terms of a.

$f(a) = a + 3\left(\frac{48}{a}\right)$

③ Graph function & find minimum:

$f(a) = a + 3\left(\frac{48}{a}\right)$

Min of 24 at $\boxed{a = 12}$

④ Find second value based on initial condition

$a \cdot b = 48$

$12 \cdot b = 48$ $\boxed{b = 4}$

2. A box with an open top is to be constructed from a square piece of cardboard, ~~3 feet wide~~, by cutting out a square(x) from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

Length = $3 - 2x$

Width = $3 - 2x$

height = $x$

Volume = $(3-2x)^2 x$

Plug into Calculator

Max of 2 at 0.5

Largest volume is $\boxed{2 ft^3}$

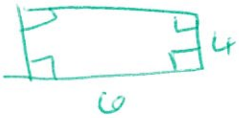
Domain: $(0, 1.5)$

What if the volume of the box is to 1.5 feet$^3$, find the value of x.

Look at graph → when is $y = 1.5 ft^3$?

$\boxed{x = 0.234 \text{ or } x = 0.826}$

3. A box with an open top is to be constructed from a rectangular piece of cardboard, 6 feet by 4 feet, by cutting out a square (x), from each of the four corners and bending up the sides. Find the largest volume that such a box can have.




Length: $6 - 2x$

Width: $4 - 2x$

Height: $x$

Volume: $(6-2x)(4-2x)x$

Plug into calculator

Max of 8.45 at $x = 0.785$

Largest volume is $\boxed{8.45 ft^3}$

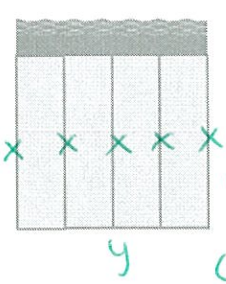
Domain: $(0, 2)$

What if the volume of the box needs to be 6 feet$^3$, find the value of x.

Look at graph (or graph $y = 6$ at the same time & find the intersection)

$\boxed{x = 0.34 \text{ or } x = 1.324}$

4. Four pig pens will be built side by side along a river by using 150 feet of fencing. What dimensions will maximize the area of the pig pens? (Hint: One is of the ~~pens~~ edges will be the river)




$x + x + x + x + x + y = 150$ ← Condition

Maximize Area $= xy$

① Rewrite condition in terms of y

$y = 150 - 5x$

② Rewrite function to maximize

$f(x) = x(150 - 5x)$

③ Graph to find max of 1125 at $x = 15$

Domain: $(0, 30)$

④ Dimensions:

$x = 15$

$y = 150 - 5(15)$

$= 75$

$\boxed{\text{Dim: } 15 \text{ by } 75}$