Building Functions Given the Roots

Example:

$x = 3, 2i$

Since they come in pairs, I know:

$x = 3, 2i, -2i$

Then $(x - 3)(x - 2i)(x + 2i)$

Multiply:

$(x - 3)(x^2 + 2i - 2i - 4i^2)$

Simplify:

$(x - 3)(x^2 + 4)$

Multiply:

$x^3 - 3x^2 + 4x - 12 = y$

1) Complex Roots come in pairs
2) Write roots as factors
3) Multiply (distribute)
4) Set equal to y

Write the equations of these polynomials if the given x values are the roots

1) $x = 2, -3, 0$

$(x-2)(x+3)(x-0)$

$(x^2 + x - 6) x$

$y = x^3 + x^2 - 6x$

2) $x = 1, 5i$

$(x-1)(x-5i)(x+5i)$

$(x-1)(x^2 + 25)$

$x^3 - x^2 + 25x - 25 = y$

3) $x = 6, 2i$

$(x-6)(x-2i)(x+2i)$

$(x-6)(x^2 + 4)$

$y = x^3 - 6x^2 + 4x - 24$

4) $x = 0, 0, -8$

$(x-0)(x-0)(x+8)$

$x^2(x+8)$

$y = x^3 + 8x^2$

5) $x = -i, 3i$

$(x-i)(x+i)(x-3i)(x+3i)$

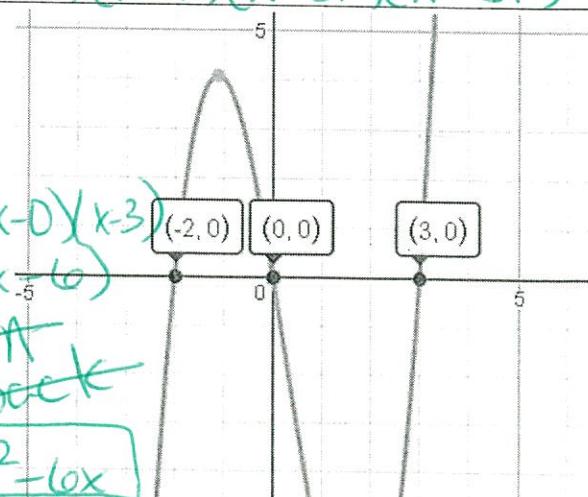$\rightarrow (x^2+1)(x^2+9) \Rightarrow (x^2+1)(x^2+9)$

$\Rightarrow y = x^4 + 10x^2 + 9$

6)

$(x+2)(x-0)(x-3)$

$x(x^2 - x - 6)$

OA back

$y = x^3 - x^2 - 6x$

(-2, 0)   (0, 0)   (3, 0)

Finding ALL Zeros/Roots of Polynomial Functions

Example:

$$3x^4 + 12x^2 = 6x^3$$

$$3x^4 - 6x^3 + 12x^2 = 0$$

$$3x^2(x^2 - 2x + 4) = 0$$

$$3x^2 = 0 \text{ and } x^2 - 2x + 4 = 0$$

$$x = \frac{-(-2) \pm \sqrt{4 - 4(1)(4)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2}$$

So $x = 0$ or $x = 1 \pm i\sqrt{3}$

1) Rewrite so P(x)=0
2) Factor
3) Zero Product Property
4) Quadratic Formula if necessary

How many roots can a polynomial have?

As many as the highest degree

Find all zeros of:

1) $x^4 = 16$

$$x^4 - 16 = 0$$

$$(x^2 - 4)(x^2 + 4) = 0$$

$$(x - 2)(x + 2)(x^2 + 4) = 0$$

$\boxed{x = 2}$ $\boxed{x = -2}$ $\quad x^2 = -4$

$\boxed{x = \pm 2i}$

2) $x^3 = 8x - 2x^2$

$$x^3 + 2x^2 - 8x = 0$$

$$x(x^2 + 2x - 8) = 0$$

$$x(x + 4)(x - 2) = 0$$

$\boxed{x = 0}$ $\boxed{x = -4}$ $\boxed{x = 2}$

3) $x^2 + 7x - 13 = 0$

QF

$$x = \frac{-7 \pm \sqrt{49 - 4(1)(-13)}}{2}$$

$$\boxed{x = \frac{-7 \pm \sqrt{101}}{2}}$$

4) $4x^2 - 25 = 0$

$$(2x - 5)(2x + 5) = 0$$

$\boxed{x = \frac{5}{2} \qquad x = \frac{-5}{2}}$

Group Swap

1) $x^3 = 64$

$x^3 - 64 = 0$

$(x-4)(x^2 + 4x + 16)$

$\boxed{x = 4}$ $\quad x^2 + 4x + 16 = 0$

$x = \dfrac{-4 \pm \sqrt{16 - 4(1)(16)}}{2}$

3) $x^4 - 5x^2 = 36$

$x^4 - 5x^2 - 36 = 0$

$(x^2 - 9)(x^2 + 4) = 0$

$(x-3)(x+3)(x^2+4) = 0$

$\boxed{x = 3}$ $\quad \boxed{x = -3}$ $\quad x^2 = -4$

$\boxed{x = \pm 2i}$

2) $x^4 - 10x^2 = -9$

$x^4 - 10x^2 + 9 = 0$

$(x^2 - 9)(x^2 - 1) = 0$

$(x-3)(x+3)(x-1)(x+1) = 0$

$\boxed{x = 3}$ $\quad \boxed{x = -3}$ $\quad \boxed{x = 1}$

$\boxed{x = -1}$

4) $x^3 + 3x^2 - x = 3$

$x^3 + 3x^2 - x - 3 = 0$

$x^2(x + 3) - 1(x + 3)$

$(x+3)(x-1)(x+1) = 0$

$\boxed{x = -3}$ $\quad \boxed{x = 1}$ $\quad \boxed{x = -1}$

Group Swap

1) $x^3 = 64$

3) $x^4 - 5x^2 = 36$

SAME AS ABOVE

2) $x^4 - 10x^2 = -9$

4) $x^3 + 3x^2 - x = 3$